Name: Date:

WORKSHEET :

Arithmetic Series

Find the summation of the following finite series.

1) 1 + 2 + 3 +....+ 99 + 100 =

2) 1 + 3 + 5 +...+ 23 + 25 =

3) 17 + 20 + 23 +...+ 56 + 59 =

4) 51 + 55 + 59 +...+ 99 + 103 =

5) 51 + 52 + 53 +...+ 99 + 100 =

6) 20 + 18 + 16 +...- 48 - 50 =

7) 25 + 20 + 15 +...- 15 - 20 - 25 =

8) -2 + 4 - 6 + 8...- 46 + 48 =

9) -50 - 45 - 40 ... + 25 + 30 =

10) 5 - 10 + 10 - 5 + 15 - 0...+ 30 + 15 =

Find the summation of the following infinite series.

11) 1 + 2 + 3 + 4 +.... =

12) -1/2 + 0 + 1/2 + 1 + 3/2 + 2 +... =

WORKSHEET:

Geometric Series

Find the summation of the following finite series.

1) 1 + 2 + 4 + 8 + 16 + 32 =

2) 1 + 3 + 9 + 27 =

3) 1/3 + 1 + 3 + 9 + 27 + 81 =

4) 5 + 25 + 625 + 3,125 + 15,625 =

5) 1/2 + 1/4 + 1/8 + 1/16 =

6) 1/3 - 1/9 + 1/27 - 1/81 =

Find the summation of the following infinite series.

9) 1/2 + 1/4 + 1/8 + 1/16... =

10) 1/3 - 1/9 + 1/27 - 1/81... =

11) 1/4 + 1/2 + 1 .... =

12) 1/27 + 1/9 + 1/3.... =

13) 8 + 2 + 1/2 + 1/8 ...=

14) 2 + 1 + 1/2 + 1/4 + 1/8... =

ANSWERS :

Arithmetic Series

Find the summation of the following finite series.

1) 1 + 2 + 3 +....+ 99 + 100
100 + 99 + 97 ..+ 2 + 1
101 + 101 + 101 ...+ 101 + 101
n = 100 terms
101 × 100/2 = 5,050

2) 1 + 3 + 5 +...+ 23 + 25 =
Sum of consecutive odds 1 to $A_n = n^2$
n = 13 terms
$13^2 = 169$

3) 17 + 20 + 23 +...+ 56 + 59 =
59 + 56 + 53 ... + 20 + 17
76 + 76 + 76 ... + 76 + 76
n = 15 terms
15 × 76/2 = 570

4) 51 + 55 + 59 +...+ 99 + 103 =
n = 14 terms, $A_1 = 51$, $A_n = 103$
$n(A_1 + A_n)/2 = 14(51 + 103)/2$
14(154)/2 = 7(154) = 1,078

5) 51 + 52 + 53 +...+ 99 + 100 =
100 + 99 + 97 ..+ 52 + 51
151 + 151 + 151 ...+ 151 + 151
n = 50 terms
151 × 50/2 = 3,775

6) 20 + 18 + 16 +...- 48 - 50 =
The terms 20 to -20 cancel.
-22 - 24 - 26 -...- 48 - 50 =
-50 - 48 - 46 -...- 24 - 22
-72 - 72 - 72....-72 - 72
n = 15 terms
15 × -72/2 = -540

7) 25 + 20 + 15 +...- 15 - 20 - 25 =
All terms cancel except 0. 0

8) -2 + 4 - 6 + 8...- 46 + 48 =
This is a sum of 2 series. Every 2 terms = +2. There are 20 terms or 10pairs so the sum is 10 × 2 = 20

9) -50 - 45 - 40 ... + 25 + 30 =
The terms 30 to -30 cancel.
-35 - 40 - 45 - 50 = -170

10) 5 - 10 + 10 - 5 + 15 - 0...+ 30 + 15 =
This is a sum of 2 increasing series. 20(6) = 120 *see tutoring for detail explanation

Find the summation of the following infinite series.

11) 1 + 2 + 3 + 4 +.... = No limit

12) -1/2 + 0 + 1/2 + 1 + 3/2 + 2 +... = No limit

ANSWERS :

Geometric Series

Find the summation of the following finite series. Sum = $A_1(1 - r_n)/(1 - r)$

1) 1 + 2 + 4 + 8 + 16 + 32 =
= $1(1 - 2^6)/(1 - 2)$ = -63/-1 = 63

2) 1 + 3 + 9 + 27 =
= $1(1 - 3^4)/(1 - 3)$ = -80/-2 = 40

3) 1/3 + 1 + 3 + 9 + 27 + 81 =
= $1/3(1 - 3^6)/(1 - 3)$ = (1/3)(-728/-2) =
364/3 = 121 1/3

4) 5 + 25 + 625 + 3,125 + 15,625 =
= $1(1 - 2^6)/(1 - 2)$ = -63/-1 = 63

5) 1/2 + 1/4 + 1/8 + 1/16 =
= $1/2(1 - 1/2^4)/(1 - 1/2)$ =
(1/2)(15/16)/(1/2) = 15/16

6) 1/3 - 1/9 + 1/27 - 1/81 =
= $(1/3)(1 - (-1/3)^4)/(1 - (-1/3))$ =
(1/3)(80/81)/(4/3) = (80/81)(1/4) = 20/81

Find the summation of the following infinite series. Sum = $A_1/(1 - r)$ if $|r| < 1$

7) 1/2 + 1/4 + 1/8 + 1/16... =
= (1/2)/(1 - 1/2) = 1

8) 1/3 - 1/9 + 1/27 - 1/81... =
= (1/3)/(1 - (-1/3)) = (1/3)(3/4) = 1/4

9) 1/4 + 1/2 + 1 .... =
r = 2 so $|r| > 1$ No limit

10) 1/27 + 1/9 + 1/3.... =
r = 3 so $|r| > 1$ No limit

11) 8 + 2 + 1/2 + 1/8 ...=
= 8/(1 - 1/4) = 8(4/3) = 32/3

12) 2 + 1 + 1/2 + 1/4 + 1/8... =
= 2/(1 - 1/2) = 2(2/1) = 4

KEY CONCEPTS:
A series is the summation of the terms in a sequence.

1. Arithmetic Series -

Finite Series:

A) Method 1 (Logical Approach):
Add the 1st term to the last then the 2nd term to the 2nd to last and so on. For an arithmetic series the sum of pairs will be the same value for every pair so the calculation becomes a multiplication of the pair sum times the number of pairs. It is the method supposedly employed by a young Carl Friedrich Gaussthat alerted his teachers of his inherent mathematical genius. There are several ways to execute this general approach only one of which (our favorite) is demonstrated below.

Step 1. Flip the order of the series and add them together as follows.

$$\begin{array}{l} 1 + \ 2 + \ 3 + \ldots + \ 99 + 100 \\ \underline{100 + 99 + 98 + \ldots + 2 + \ 1} \\ 101 + 101 + 101 + \ldots + 101 + 101 \end{array}$$

Step 2. Multiply the pair total (101 in this case) by the number of terms, n. The number of terms can often be evaluated by counting directly or by the arithmetic sequence formula from last lesson; $A_n = A_1 + d(n - 1)$.

101×100

Step 3. Divide the result by 2 because the series was summed twice.

$101 \times 100/2 = 101 \times 50 = 5{,}050$

This method works in general regardless of the value of A_1 or d.

B) Method 2 (Formulaic Approach):

The formulas can be derived in several ways, but one approach is to use the average of a symmetrical sequence where the mean = median and that is equivalent to $(A_1 + A_n)/2$, the average of the 1st and last term. The count is the number of terms, n. The sum can be derived from the average.

Sum = Count Å~ Average = $\mathbf{n(A_1 + A_n)/2}$

In the above case that equates to the same result.

$100 \times (1 + 100)/2 = 50 \times 101 = 5{,}050$

There are specific cases that may also come in handy.

1. If $d = 1$ and $A_1 = 1$ the series is the sum of consecutive integers from 1 to n. In this case $A_n = n$ and the formula becomes...

Sum of Integers 1 to n = $\mathbf{n(n + 1)/2}$

2. Sum of Odd Integers 1 to A_n = $\mathbf{n^2}$

This is known also as a square sequence. e.g.

$1 + 3 = 4 = 2^2$
$1 + 3 + 5 = 9 = 3^2$
$1 + 3 + 5 + 7 = 16 = 4^2$
$1 + 3 + 5 + 7 + 9 = 25 = 5^2$ ....

Infinite Series:

The sum of an arithmetic sequence has no limiting value.

* Note the names given to Method 1 and Method 2 above are merely descriptive.

2. Geometric Series -

Finite Series:

Sum = $\mathbf{A_1(1 - r^n)/(1 - r)}$

where r is the common multiplier, A_1 the 1st term, and n the number of terms.
e.g. $1 + 2 + 4 + 8 + 16 = 31$
$A_1 = 1$, $n = 5$, $r = 2$ so the sum = $1(1 - 2^5)/1 - 2) = (1 - 32)/(1 - 2) = -31/-1 = 31$

Infinite Series:

Infinite series **only approach a limit if |r| < 1** where r is the common multiplier.

In that case $r^n \rightarrow 0$ as n approaches infinity for the summation above. As a result,

Sum = $A_1(1 - 0)/(1 - r) = A_1/(1 - r)$

e.g. $1 + 1/2 + 1/4 + 1/8 + \ldots = 1/(1 - 1/2) = 1/(1/2) = 1 \times (2/1) = 2$
The infinite summation where $r = 1/2$ (i.e. $|r| < 1$) is a finite value; 2