Answer the following questions.

1) 15 boys and 10 girls apply to go on a school trip. Three students are selected at random. The probability that 1 girl and 2 boys are selected, is:

2) A bag contains 4 white, 5 red and 6 green balls. Three balls are drawn at random from the bag. The probability that all of them are green, is:

3) Two cards are drawn together from a pack of 52 cards. What is the probability that one card is clubs and one card is spades?

4) Two cards are drawn together from a pack of 52 cards. What is the probability that both cards have a red colored suit?

5) Three unbiased coins are tossed. What is the probability of getting at most one tail?

6) How many even 4 digit numbers are there?

7) Jane and John are preparing for the SHSAT. Jane gets math problems correct 90% of the time and John gets them correct 70% of the time. What is the probability that exactly one of them gets any given problem correct?

8) Two numbers are selected from a hat containing 20 numbers 1 through 20. What is the probability that one number is not twice the other number selected?
1) 15 boys and 10 girls apply to go on a school trip. Three students are selected at random. The probability that 1 girl and 2 boys are selected, is: \[ P(E) = \frac{C(15,1) \times C(10,2)}{C(25,3)} = \frac{21}{46} \]

2) A bag contains 4 white, 5 red and 6 green balls. Three balls are drawn at random from the bag. The probability that all of them are green, is: \[ P(E) = \frac{C(6,3)}{C(15,3)} = \frac{4}{91} \]

3) Two cards are drawn together from a pack of 52 cards. What is the probability that one card is clubs and one card is spades? \[ P(E) = \frac{C(13,1) \times C(13,1)}{C(52,2)} = \frac{13}{102} \]

4) Two cards are drawn together from a pack of 52 cards. What is the probability that both cards have a red colored suit? \[ P(E) = \frac{C(26,2)}{C(52,2)} = \frac{25}{102} \]

5) Three unbiased coins are tossed. What is the probability of getting at most one tail? \[ P(E) = \frac{C(3,0) + C(3,1)}{2^3} = \frac{4}{8} = \frac{1}{2} \]

6) How many even 4 digit numbers are there? 4,500

7) Jane and John are preparing for the SHSAT. Jane gets math problems correct 90% of the time and John gets them correct 70% of the time. What is the probability that exactly one of them gets any given problem correct? The sample space has 4 possible outcomes: Jane and John get it correct, Jane gets it correct and John gets it incorrect, John gets it correct and Jane gets it incorrect, Jane and John get it incorrect

Calculate the probability of each scenario.
90% \times 70% = 63%
90% \times (100% - 70%) = 90% \times 30% = 27%
(100% - 90%) \times 70% = 10% \times 70% = 7%
(100% - 90%) \times (100% - 70%) = 10% \times 30% = 3%
In scenario 2 and 3 "exactly one" gets a question correct. The sum of the two is

\[ P(\text{exactly 1}) = 27% + 7% = 34% \]

8) Two numbers are selected from a hat containing 20 numbers 1 through 20. What is the probability that one number is not twice the other number selected?
\[ P(\text{1 is twice the other}) = \frac{2 \times 10}{\binom{20}{2}} = \frac{20}{20} \times (\frac{1}{19}) = \frac{1}{19} \] likelihood one number is twice the other.
Be careful, the question asks for the probability that one number is not twice the other. That will be the complement.

\[ P(\text{1 is not twice the other}) = 1 - P(\text{1 is twice the other}) = 1 - \frac{1}{19} = \frac{18}{19} \]
KEY CONCEPTS:
Probability (More Difficult).

1. Counting Correct Outcomes in Numerator & Denominator -

Many students try to work with probabilities right from the start rather than leverage their instruction on ways to count and the definition of probability; the count of successful outcomes divided by total outcomes. Although that will work if properly done, many students, for example, will learn 1/8 is the probability of getting all heads on 3 coin flips or that 7/8 is the probability of getting at least one head. They do not, however, incorporate the count of arrangements when asked what is the chance of exactly one head. Again, 1/8 is a typical response when there are 3 ways to arrange 1 tail out of 3 coin tosses. The correct probability of exactly 1 head is 3/8. Developing the correct counts is perhaps the best way to avoid this mistake.

The counts for the numerators (number of ways to generate successes) is perhaps the more challenging half of the probability calculation, but it can be easier than students think. Often the combinations formula can be utilized to good effect and it is worth practicing its application for numerator and denominator in many cases.

2. "AND" versus "OR" -

The union and intersection of events (essentially "or" and "and" conditions in word problems) is an important distinction that can result in greater success if learned or more confusion if not considered formally. Many problems in this section guide students to practice both.

\[ P(E \text{ or } F) = P(E) + P(F) \quad \text{assuming E and F are mutually exclusive} \]

\[ P(E \text{ and } F) = P(E) \times P(F) \]