

Name:

Date:

WORKSHEET :



**Triangle Inequality
Theorem**

Triangle Inequality Theorem

State if each set of three numbers can be the lengths of the sides of a triangle.

1) 7, 2, 11

6) 5, 12, 12

2) 3, 5, 10

7) 11, 10, 8

3) 13, 5, 11

8) 9, 12, 24

4) 2, 13, 20

9) 4, 10, 16

5) 5, 2, 8

10) 2, 6, 13

Given are the lengths of two sides of a triangle. Find the range of lengths for the third side.

11) 10, 13

14) 9, 7

12) 5, 12

15) 4, 3

13) 11, 4

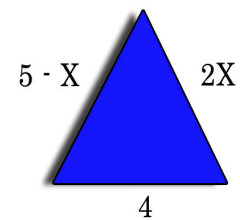
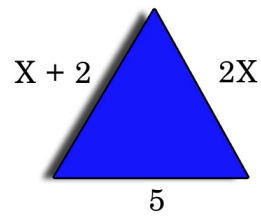
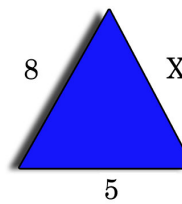
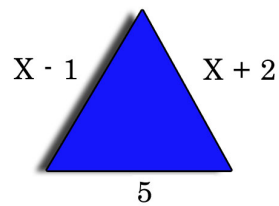
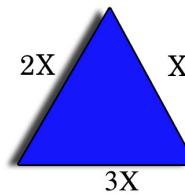
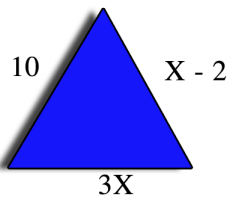
16) 2, 7

WORKSHEET :



Triangle Inequality
Theorem

Solve for x: Use Triangle Inequality theorem ($a < b + c$, $b < a + c$, $c < a + b$) to solve.



ANSWERS :



Triangle Inequality Theorem

Triangle Inequality Theorem

State if each set of three numbers can be the lengths of the sides of a triangle.

1) 7, 2, 11

No

6) 5, 12, 12

Yes

2) 3, 5, 10

No

7) 11, 10, 8

Yes

3) 13, 5, 11

Yes

8) 9, 12, 24

No

4) 2, 13, 20

No

9) 4, 10, 16

No

5) 5, 2, 8

No

10) 2, 6, 13

No

Given are the lengths of two sides of a triangle. Find the range of lengths for the third side.

11) 10, 13

$$3 < x < 23$$

14) 9, 7

$$2 < x < 16$$

12) 5, 12

$$7 < x < 17$$

15) 4, 3

$$1 < x < 7$$

13) 11, 4

$$7 < x < 15$$

16) 2, 7

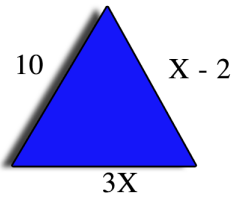
$$5 < x < 9$$

ANSWERS :

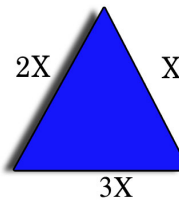


Triangle Inequality Theorem

Solve for x: Use Triangle Inequality theorem ($a < b + c$, $b < a + c$, $c < a + b$) to solve.

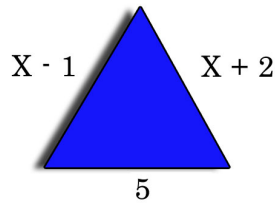


$$\begin{aligned} 10 < 3x + x - 2 & \quad 12 < 4x & \quad 3 < x \\ 3x < 10 + x - 2 & \quad 2x < 8 & \quad x < 4 \\ x - 2 < 10 + 3x & \quad -12 < 2x & \quad 6 < -6 \\ 3 < x < 4 \end{aligned}$$

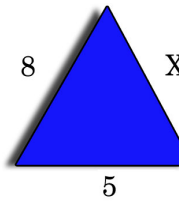


$$\begin{aligned} 2x < 3x + x & \quad 2x < 4x & \quad 2 < 4 \text{ ALWAYS TRUE} \\ 3x < 2x + x & \quad 3x < 3x & \quad 3 < 3 \text{ NOT TRUE} \\ x < 2x + 3x & \quad x < 5x & \quad 1 < 5 \text{ ALWAYS TRUE} \end{aligned}$$

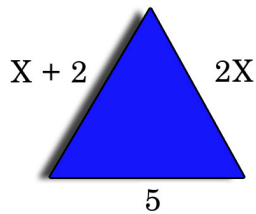
Triangle cannot exist



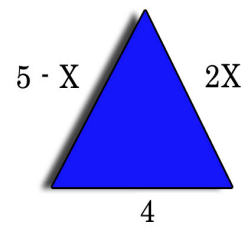
$$\begin{aligned} 5 < x - 1 + x + 2 & \quad 5 < 2x + 1 & \quad x > 2 \\ x - 1 < 5 + x + 2 & \quad -1 < 7 & \quad \text{ALWAYS TRUE} \\ x + 2 < 5 + x - 1 & \quad 2 < 4 & \quad \text{ALWAYS TRUE} \\ x > 2 \end{aligned}$$



$$\begin{aligned} 5 < x + 8 & \quad -3 < x & \quad x > 0 \text{ already true} \\ 8 < 5 + x & \quad x > 3 \\ x < 5 + 8 & \quad x < 13 \\ 3 < x < 13 \end{aligned}$$



$$\begin{aligned} 5 < 2x + x + 2 & \quad 3 < 3x & \quad 1 < x \\ 2x < 5 + x + 2 & \quad x < 7 \\ x + 2 < 5 + 2x & \quad -3 < x & \quad x > 0 \text{ already true} \\ 1 < x < 7 \end{aligned}$$



$$\begin{aligned} 4 < 5 - x + 2x & \quad -1 < x & \quad x > 0 \text{ already true} \\ 2x < 4 + 5 - x & \quad x < 9 \\ 5 - x < 4 + 2x & \quad 1 < x \\ 1 < x < 9 \end{aligned}$$

KEY CONCEPTS:

The Triangle Inequality Theorem is a test to see if the triangle can exist or not.

1. **Any one side of a triangle must be less than the sum of the other two sides.**

3 Checks		Equivalent "Greater than positive difference" check
$a < b + c$		
$b < a + c$	\rightarrow	$a > b - c$
$c < a + b$	\rightarrow	$a > c - b$

2. The result of any check of all 3 inequalities could be 0, 1, 2 constraints or the triangle does not exist.

3. If any one inequality results in NOT TRUE then the triangle DOES NOT EXIST

4. If any one inequality results in ALWAYS TRUE then move on to check the other conditions as the given condition provides no constraints.

5. Any one side length must be greater than 0. i.e. $x > -5$ is not a constraint because $x > 0$ is already more restrictive.

6. Students often learn to test if side a is less than $b + c$ and then if a is greater than the positive difference of b and c rather than all 3 check, but in many cases the positive difference cannot be determined. e.g. $a = 5$ and $b = 2z$ and $c = z + 5$. Is $2z$ greater than $z + 5$?

* In order to be safe it is good to get in the habit of making all three checks unless the "positive" difference between the other sides is certain.

7. ** Like any equation and inequality plug your results back into the original conditions to make sure the numerical results make sense.

TYPICAL MISTAKES TO WATCH FOR:

1. Be careful about the direction of the inequality symbol. Many problems put the variable on the right side and force you to switch the entire inequality.

e.g. $5 > y$ is not $y > 5$. It is $y < 5$

2. Do not forget to change the direction of the inequality symbol if dividing by a negative!

e.g. $-y < -5$ becomes $y > 5$

3. The inequality theorem conditions are "less than" only and NEVER include "equal than"